Is Indeterminate Identity Incoherent?

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In "Counting and Indeterminate Identity" (Pinillos, 2003), N. Ángel Pinillos develops an argument that there can be no cases of 'Split Indeterminate Identity' (henceforth, SII). Such a case would be one in which it was indeterminate whether $a = b$ and indeterminate whether $a = c$, but determinately true that $b \neq c$. An putative example would be the Ship of Theseus: If we dub the original ship a and the ships that return to port b and c, there is at least some reason to think we have a case of SII, and some philosophers, for example, Terrence Parsons, have defended just this view (**?**).

Pinillos's argument is as follows. (My exposition of his argument differs somewhat from his presentation of it, but not in ways that should matter.) He argues first that just one ship left port and exactly two ships returned. If so, then either $a \neq b$ or $a \neq c$. He then presents a formal argument that this last claim—that either $a \neq b$ or $a \neq c$—is inconsistent with the assumption that it is indeterminate whether $a = b$ and indeterminate whether $a = c$. The interest of the argument lies, in part, in the fact that it appears to appeal to none of the controversial claims to which similar arguments due to Gareth Evans (Evans, 1985) and Nathan Salmon (**?**) appeal.

In this note, I will argue for two counter-claims. First, the formal argument fails to establish its conclusion, for essentially the same reason Evans's and Salmon's arguments fail to establish their conclusions. Second, the phenomena in which Pinillos is interested, which concern the cardinalities of sets of vague objects, manifest the existence of what Kit Fine called 'penumbral connections', phenomena that the logics Pinillos considers are already known not to handle well. At best, then, his argument simply illustrates those same flaws in a new way. My goal here is thus not to defend Parsons's treatment of indeterminate identities but to show that Pinillos's arguments, if they work at all, work only against certain forms of the view that there can be cases of split indeterminate identity.

1 The Formal Argument

The formal argument as Pinillos presents it begins with a set-theoretic argument for the claim that either $a \neq b$ or $a \neq c$ (Pinillos, 2003, pp. 37–8). I will not raise questions about this part of the argument. My interest is in Pinillos's purported derivation of a contradiction from this disjunction and the premises that are distinctive of a case of SII: That it is indeterminate whether $a = b$ and also whether $a = c$.

There is an odd change when one reaches this part of Pinillos's argument. Whereas, in the preceding portion, one had formulae occurring in the derivation, here one instead finds meta-theoretic claims about what entails what. The reason is that Pinillos needs to appeal, in this part of his argument, to the claim that from the claim that P one may infer that it is not indeterminate whether P. As he notes, it is now well known that the validity of this inference needs to be distinguished from the validity (or even truth) of the associated conditional "If P, then it is not indeterminate whether P". Natural systems that allow the mentioned inference may invalidate the associated conditional.[1]

Call the rule in question DET and write '∇A' to mean: it is indeterminate whether A. Then we may represent the relevant part of Pinillos's argument as follows:

(1)	$a \neq b \vdash \neg\nabla a \neq b$	DET
(2)	$a \neq b \vdash \neg[\nabla(a \neq b)\&\nabla(a \neq c)]$	TF (1)
(3)	$a \neq c \vdash \neg\nabla a \neq c$	DET
(4)	$a \neq c \vdash \neg[\nabla a \neq b\&\nabla(a \neq c)]$	TF (3)
(5)	$a \neq b \vee a \neq c \vdash \neg[\nabla(a \neq b)\&\nabla(a \neq c)]$	Proof by Cases (2,4)

The argument is somewhat different from Pinillos's (Pinillos, 2003, p. 39), though it is to the same effect. Lines (1) and (3) of this argument correspond to lines (22) and (23) of his. At line (24) of his, he says that each of these 'entails something that contradicts' the claim that $\nabla(a \neq b)\&\nabla(a \neq c)$. Lines (2) and (4) of the present argument capture that sentiment. On lines (25) and (26), Pinillos claims that we have a 'contradiction by cases' from which it follows that '$\nabla(a \neq b)\&\nabla(a \neq c)$' cannot be true. That claim corresponds to line (5) of the present argument, though it may not be quite what Pinillos intends. The extremely informal character of his argument makes it difficult to be sure.

The crucial step in the argument lies in the appeal to proof by cases at line (5). Now, as said above, it is essential to distinguish the claim that the inference DET is valid from the claim that all instances of the conditional '$A \rightarrow \neg\nabla A$' are true. And to enforce that distinction, one must disallow appeal to DET within the context of conditional proofs. Otherwise, one could derive any instance of the mentioned conditional as follows:

(1)	$A \vdash \neg\nabla A$	DET
(2)	$\vdash A \rightarrow \neg\nabla A$	Conditional proof (1)

[1] For one such system, see (Heck, 1998).

There are similar questions to be raised about Pinillos's appeal to DET within a proof by cases. If one allows such an appeal, then one can argue as follows:

(1)	$A \vdash \neg\nabla A$	DET
(2)	$\neg A \vdash \neg\nabla\neg A$	DET
(3)	$\neg A \vdash \neg\nabla A$	(see text)
(4)	$A \vee \neg A \vdash \neg\nabla A$	Proof by cases

Line (3) here is justified by the fact that its being indeterminate whether ¬A just is its being indeterminate whether A. Now, if we do not have the law of excluded middle, we cannot prove that it is not indeterminate whether A outright. But some of us do affirm excluded middle. Moreover, a defender of SII who did not wish to do so might yet object that it is not *contradictory* to hold that either A or not-A but it is indeterminate which. But it would be if such applications of DET were permitted. At the very least, then, it is far from clear that Pinillos can legitimately appeal to DET where he does.

Moreover, one can show that, in a logic that allows the application of DET only outside subordinate deductions (in particular, which does not allow it within conditional proof, proof by cases, and *reductio*), no such conclusion as that Pinillos seeks can be reached *unless one assumes that formulae involving* '∇' *can appear in applications of Leibniz's Law*. But this assumption is the crucial one for Evans's and Salmon's arguments. If we allow it, then we already know that '$\nabla(a \neq b)$' cannot be true if DET is a valid rule. But if we disallow the assumption, then '$\nabla(a \neq b)$' can be true. Pinillos's argument is therefore an improvement over Evans's and Salmon's only if it succeeds without this assumption. But it does not: A model for a logic that validates DET, sustains classical logic, and verifies both '$\nabla(a \neq b) \& \nabla(a \neq c)$' and '$b \neq c$', is easily constructed by adapting the construction in (Heck, 1998).

2 The Informal Argument

Here's what I think is really bothering Pinillos. Call the ship that left port *Orig*(inal) and the two that returned *Spat*(ially continuous) and *Part*(s are the same). We would have a case of SII if it were indeterminate whether *Spat* were *Orig* and whether *Part* were *Orig* but definitely true that *Spat* were not *Part*. How many ships, then, left port? The answer would seem to be 'one'. But if it is indeterminate whether *Spat* is *Orig*, then it is presumably indeterminate whether *Spat* left port, only later to return, or instead came into existence at some point along the way. And Pinillos finds the following principle

(PCI) If it is indeterminate whether x has property P, then there is no determinate answer to the question how many things have property P.

appealing. It is, indeed, a principle he thinks "has great plausibility regardless of what stance one takes towards vague identity or SII" (Pinillos, 2003, p. 39). But if it is indeterminate whether *Spat* is *Orig*, then it is indeterminate whether *Spat* left port, and PCI entails that it is indeterminate how many ships left port, whence it is *not* be correct to say that just one ship left port.

Pinillos is inclined to blame the problem here on SII, but it is PCI that is at fault. PCI is appealing when one considers a single object in isolation: Here is Jones, and it's indeterminate whether he's bald. If so, then surely it is also indeterminate how many bald people there are, since if Jones is bald, that's one more, and if he's not, that's one fewer. Similarly, one might think that, if it is indeterminate whether *Spat* left port, then it must be indeterminate how many ships left port: If *Spat* did, that would be one more; if not, one less. But that's wrong. The case of *Spat* is different from the case of Jones. *Orig* left port. If *Spat* did, then that is because *Spat* is identical to *Orig*, and then *Spat* is not one *more* ship that left but just *the* ship that left. The same goes for *Part*.

One might worry, similarly, that if it is indeterminate whether *Spat* left port and also indeterminate whether *Part* did, then maybe both *Spat* and *Part* left, in which case two ships would have left. But so to reason would be to suppose that it is indeterminate whether both *Spat* and *Part* left port, and it is not: It is (definitely) true that not both *Spat* and *Part* left port. Only *Orig* did, and it is definitely true that not both *Spat* and *Part* are *Orig*.

Pinillos himself argues that, if SII holds, then it is not false that (and so it is indeterminate whether) exactly one ship *returned* to port (Pinillos, 2003, pp. 44ff). Exactly one ship returned if and only if there is a ship that returned and nothing else did. But consider *Orig*. Certainly *Orig* returned. At the very least, it is not false that it did. But nor is it false that nothing else did. After all, it is false neither that *Spat* is identical to *Orig* nor that *Part* is identical to *Orig*. So both these options are open, which leaves the possibility open that Orig returned and nothing else did. But the problem, here again, is that the indeterminacies at issue are not independent of one another. It is indeed false neither that *Spat* is identical to *Orig* nor that *Part* is. But these indeterminacies cannot both be resolved the same way. If *Spat* is identical to *Orig*, then *Part* is not, and then two ships will have returned: *Spat* (that is, *Orig*) and *Part*. If *Spat* is not identical to *Orig*, then *Part* is, and so again two ships will have returned: *Part* (that is, *Orig*) and Spat. No contradiction is forthcoming without stronger principles.

Pinillos in fact employs such stronger principles, in the form of assumptions about the semantics governing the operator '∇' and, indeed, the logical connectives. The assumption, in brief, is that the underlying semantics is many-valued. Parsons, who is Pinillos's main target, does of course make this assumption. But if that is what causes the trouble, then that is what causes the trouble. In the sorts of many-valued systems under discussion, one simply cannot make good sense of the 'dependencies among indeterminacies' we have been discussing, any more than one can make good sense in such systems of the fact that anyone taller than someone who is tall must be tall. Such statements of 'penumbral connections', as Kit Fine called them (Fine, 1975), are counted indeterminate by many-valued systems.

Supervaluational approaches, on the other hand, are designed to accommodate just such connections between indeterminacies, and an approach to indeterminate identities based upon a broadly supervaluational framework handles cardinality judgements smoothly, as a little experimentation will show. Pinillos explicitly declines to offer an argument against such approaches, on the ground that "...the indeterminacy in question is due to the world as opposed to an imprecision in the language used to describe the situation..." (Pinillos, 2003, p. 37).

But I detect a missing premise, namely, that formal treatments that depend upon the use of a broadly supervaluational semantics necessarily treat indeterminacies as linguistic rather than worldly. Though some defenders of supervaluational approaches accept that claim, and Fine's original motivation for the approach certainly employed it, I know of no general argument for this claim and don't find it particularly obvious in itself. More to the point, however, in the context of Pinillos's argument, the question we ought to be asking is this one: Is there reason to suppose that someone committed to SII is committed to denying the existence of the sorts of penumbral connections to which we need to appeal if we are to endorse the judgements of cardinality that Pinillos rightly tags as most plausible? I know of no such reason.

References

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